Photonic defect modes of cholesteric liquid crystals

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We investigate defect modes of cholesteric liquid crystals as photonic band gap materials. For normal incidence of light, cholesteric liquid crystals exhibit total reflection for the circular polarization with the same handedness as that of cholesteric helix. However, the other orthogonal component is completely transmitted. When we replace a thin layer of liquid crystal by an isotropic material as a defect, defect modes are induced for both polarizations of incident light. We analyze the wavelength and reflectivity of the defect modes in terms of the refractive index of defect layer. [S1063-651X(99)09312-5]

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Systematic studies of light propagation in a medium with a spatial periodic modulation of dielectrics have been carried out widely. As in the electronic bands of a solid, a periodic array of dielectrics forms forbidden photonic bands which are called photonic band gaps (PBG's) [1,2]. But, in general, when $\mu \neq 1$, the wave impedance is the relevant quantity instead of the dielectric constant for PBG's to exist [3]. In photonic crystals, defects can be introduced by replacing a part of the host medium with a material that has a different dielectric constant, and can make defect modes in the forbidden bands. It was shown that the origin of defect modes is the phase change due to the change in the optical path length caused by the defect medium [4]. Recently, interest in synthesizing PBG materials that can be spontaneously formed has been raised. Studies of self-assembled colloidal particles, nanoparticles, or nanostructured films are good examples [5–7].

Here, we theoretically study the reflection and transmission of light from a one-dimensional system consisting of two layers of cholesteric liquid crystal (ChLC) sandwiching a thin layer of isotropic medium (see Fig. 1). ChLC is an organic self-assembled material. It has one-dimensional periodic modulation of dielectric constant and a stop band in visible range. The spatial period can be easily varied by adding dopants and changing temperature.

ChLC is composed of optically anisotropic and uniaxial elements [8]. One major and two minor principal axes are mutually perpendicular locally and the corresponding dielectric constants are $\epsilon_{\parallel}$ and $\epsilon_{\perp}$. The major and one minor principal axes are helically twisted along the other minor axis retaining their orthogonality. This way, ChLC has one-dimensional periodic modulation of dielectric constants. The pitch P_o is the period of helical twist, but the true period of dielectric tensor function is $P_o/2$, not P_o, by the head-tail symmetry of liquid crystals. Owing to the optical anisotropy and helical twist, ChLC has an interesting reflection property which a conventional quater-wave stack does not; this is the selective reflection. For the light incident along the helical axis, the circularly polarized light with the same handedness as that of ChLC is totally reflected in the frequency range of $|q_o|/n_e < \omega/c < |q_o|/n_o$. Here, $n_e = \sqrt{\epsilon_{\parallel}}$ is the extraordinary index of refraction, $n_o = \sqrt{\epsilon_{\perp}}$ the ordinary one, and $q_o = (\pm)2\pi/P_o$, where $(\pm)$ represents the handedness of ChLC. It was shown that ChLC has, in general, two propagation modes, but only one mode is allowed in the frequency range given above [9]. This mode is circularly polarized with the opposite handedness to that of ChLC. In the strict sense, ChLC is not a PBG material because the stop band exists only for one circular polarization and that is caused by the optical anisotropy. In this respect, we may expect that optically anisotropic systems have new interesting phenomena that isotropic ones do not.

In this paper, we investigate defect modes of ChLC that come about when one part of ChLC is replaced by an optically isotropic material as a defect. Reflection spectra for normal incidence is studied to obtain the wavelengths and

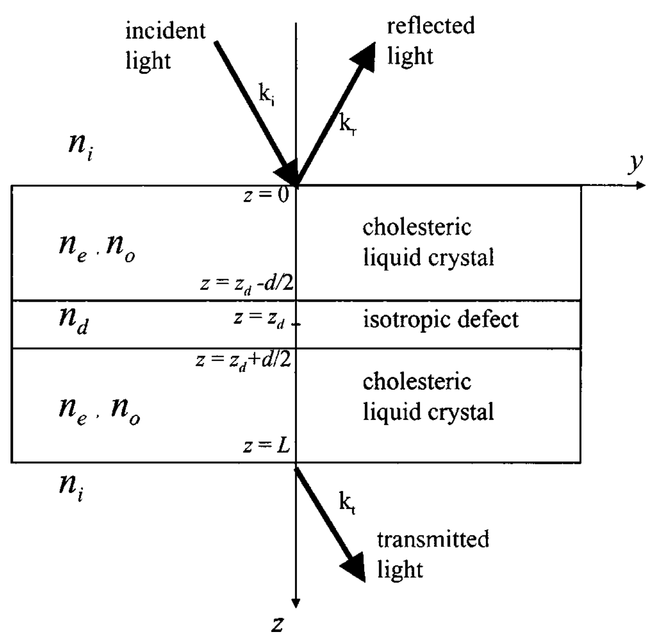

FIG. 1. Schematic drawing of liquid crystal cell used in the calculation for normal incidence. The cell is filled with cholesteric liquid crystal except for the defect layer. Values of the parameters used in this work are listed in Table I.

TABLE I. Values of the parameters used in the calculation.

	Symbol	Value
refractive index of surroundings of cell	n_i	1.6
extraordinary refractive index	$n_e=\sqrt{\epsilon_\parallel}$	1.7
ordinary refractive index	$n_o=\sqrt{\epsilon_\perp}$	1.5
pitch of ChLC	P_o	0.345 μm
maximum wavelength of band	n_eP_o	588 nm
minimum wavelength of band	n_oP_o	518 nm
cell gap	L	$21P_o$
thickness of defect layer	d	$(200/256)P_o$
thickness of artificial slab		$P_o/256$
position of defect	z_d	$10.5P_o$
wavelength range	λ_o	400–700 nm
incident angle	(θ,ϕ)	(0.0, 0.0)

intensities of defect modes when refractive index and thickness of defect layer are varied. Following the 4×4 Berreman numerical method, the equation of light propagation with angular frequency ω is given by [10]

$$\frac{d\Psi(z)}{dz}=i\frac{\omega}{c}\mathbf{\Delta}(z)\Psi(z), \tag{1}$$

where $\Psi(z)=(E_x,H_y,E_y,-H_x)^T$. $\mathbf{\Delta}(z)$ is Berreman matrix which depends on both the dielectric function and the incident wave vector. If $\mathbf{\Delta}$ is constant in z, the solution of the above equation is

$$\Psi(z)=\exp\left[i\frac{\omega}{c}\mathbf{\Delta}z\right]\Psi(0). \tag{2}$$

Equation (2) is used to get the approximate solution of Eq. (1). ChLC is virtually sliced into a large number of thin slabs. Because $\mathbf{\Delta}$ can be treated as a constant in every slab, one calculates transfer matrix with Eq. (2). According to the Cayley-Hamilton theorem [11], Eq. (2) can be expressed by a third-order equation in $\mathbf{\Delta}$, and then the thicker slab can be chosen, still achieving the same degree of accuracy which one usually obtains in the truncated Taylor expansion method [12]. This so-called ''faster Berreman method'' [13,14], whose results agree well with experimental data [15], is thus employed for this study.

Figure 1 depicts the cell structure and the coordinate system. ChLC is distributed over the cell except for the region where the defect is present. We choose the z axis to be the twist axis of ChLC. The handedness of the host ChLC is assumed to be right. The liquid crystal director is represented by $\hat{\mathbf{n}}=[\cos(q_oz),\sin(q_oz),0]$, and its dielectric tensor function $\epsilon(\mathbf{z})$ by $\epsilon_{ij}=\epsilon_\perp\delta_{ij}+\Delta\epsilon n_in_j$. Here, $(i,j)\in\{x,y,z\}$, δ_{ij} is the Kronecker delta, and $\Delta\epsilon=(\epsilon_\parallel-\epsilon_\perp)$ [8]. We only consider here the case of $\mu=1$ since we are dealing with optically visible region. It should be noted that the defect layer is located at the center of the cell, i.e., $z_d=L/2$. The parameters used in this study are summarized in Table I.

Figure 2 shows the reflection spectra of ChLC cell for each polarization of incident wave when the defect layer is introduced. R_R and R_L mean, respectively, the reflectivity of the light of right circular polarization (RCP) and of left circular polarization (LCP). The incident light of RCP is reflected, while that of LCP is transmitted, in the wavelength region of $n_oP_o<\lambda_o<n_eP_o$ (frequency range of $q_o/n_e<\omega/c<q_o/n_o$ with $q_o=2\pi/P_o$). This region behaves as a photonic band gap for the incident wave of RCP which is the assumed handedness of ChLC in the cell. As one can see in the figure, a sharp dip for R_R and a peak for R_L are induced by the introduction of a defect layer; each defect mode propagates through the corresponding stop gap. As shown in the inset, the defect mode for RCP and that for LCP have the same wavelength with the same value of reflectivity at the center of the peaks.

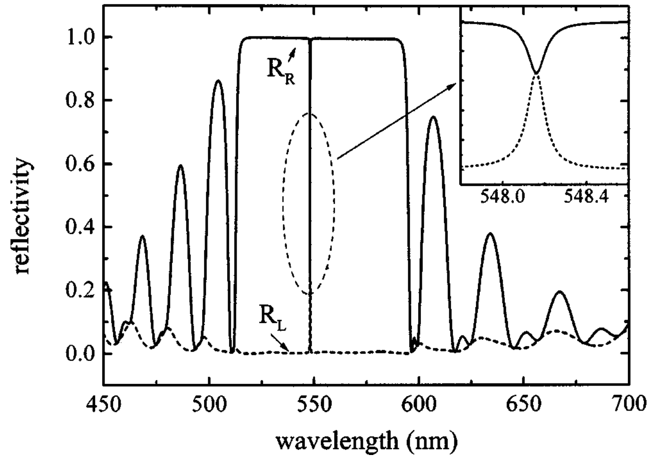

FIG. 2. Reflection spectra for incident waves of right and left circular polarization. R_R (solid) and R_L (dotted) represents the reflectivity for incident polarization of RCP and LCP, respectively. Refractive index of defect layer n_d is 2.04.

Figure 3 shows the behavior of defect modes as a function of the refractive index of defect layer n_d. Defect modes have donor- (acceptor-)like behavior when n_d is greater (less) than that of the host ChLC, and their wavelengths increase from the minimum to the maximum value of a forbidden gap as n_d increases. However, two defect modes momentarily appear near both ends of the gap (e.g., when n_d is around 2.55), then the longer-wavelength mode leaves out of and the other shorter-wavelength mode moves into the forbidden gap as n_d increases further. The wavelength of defect mode λ_d varies linearly with n_d near the center of the gap but deviates slightly near band edges. All these behaviors repeat in sequence as n_d increases. If the thickness of defect layer d gets

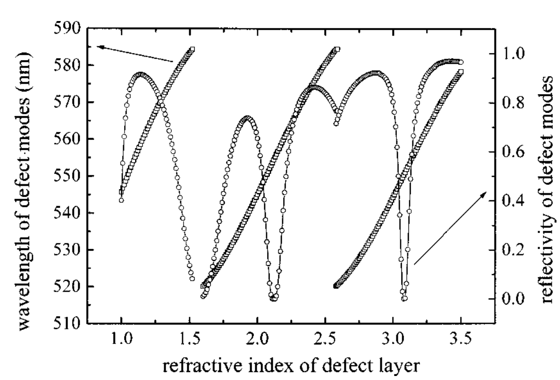

FIG. 3. Wavelengths (square) and reflectivity (circle) of defect modes as a function of the refractive index of the defect layer.

larger (smaller), the slope of λ_d vs n_d graph becomes steeper (flatter).

We find that when the defect mode exists at the center of forbidden gap, n_d satisfies the relation

$$|n_d-\overline{n}|d=\left(m+\frac{1}{2}\right)\frac{\lambda_d}{2}=\left(m+\frac{1}{2}\right)\frac{\overline{n}P_o}{2}, \tag{3}$$

where $m=0,1,2,\ldots$, and $\lambda_d=P_o(n_e+n_o)/2=P_o\overline{n}$, so that the effective refractive index of ChLC may be taken as $\overline{n}$. Then, the phase slip due to the defect layer is $(2\pi/\lambda_d)(n_d-\overline{n})d$. Equation (3) tells us that the phase slip is odd multiples of $\pi/2$, i.e., a quarter wavelength. It is well known that the phase slip of $\pi/2$ in a quarter-wave stack makes a defect level at the center of the forbidden gap [4].

We can also notice in Fig. 3 that the reflectivity becomes zero at certain values of n_d. It is found to occur when the condition

$$n_d d=\frac{\lambda_d}{2}l \tag{4}$$

is satisfied. Here, $l=1,2,3,\ldots$. This is the condition for the complete transmittance through a thin film [16]. So, the lights passing through a defect layer are not reflected, but only get the phase factor which results in defect modes. If there is a partial reflection from the defect layer, the reflectivity of such λ_d does not vanish.

Since no forbidden gap exists for the light of LCP, we cannot apply the concept of PBG, in the strict sense, to this case. Nevertheless, we can arrange two ChLC cells of opposite handedness in series at a distance much longer than the coherence length of light. They behave as a PBG material. In some cases, ChLC is more useful than conventional PBG materials. ChLC is a good circular polarizer in wide spectral ranges. One can use either the transmitted or the reflected light at one's convenience. Defect modes can have extremely narrow spectral width in the order of few Å.

In conclusion, we studied photonic defect modes of cholesteric liquid crystals as a new type of material giving photonic band gaps. ChLC exhibits such stop gaps that no propagation mode is allowed for one polarization, but the other orthogonal polarization propagates through. The light of circular polarization with the same handedness as that of ChLC is forbidden. A defect layer introduces defect modes not only for the forbidden polarization, but also for the allowed polarization. Their wavelengths and reflectivity are the same at the centers of the two defect peaks when a defect layer is located at the center of the cell.

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